cmi **TESSELLATE PRESENTS**

Scholastic Test of Excellence in Mathematical Sciences

(S.T.E.M.S.)

Title Sponsor

BRILLIANT

Math and science done right

Computer Science Exam

School Category

January 13, 2018

Message from The S.T.E.M.S. Team

Dear applicant,

Greetings from team Tessellate. This is the first edition of Scholastic Test of Excellence in Mathematical Sciences. All that we have to say before the exam is: DO NOT BE NERVOUS! The questions can be too hard for some of you, and too easy for the rest. Remember, the aim should not be to solve every question completely. Rather, try to attempt the questions, and write whatever you are thinking, even if it is a couple of lines. If you think that you are not being able to solve the questions, and they are too hard, well then they will be hard for everyone and the ranking will be relative. We are NOT looking for exactly correct answers for all the questions! Rather, we would like to see what you are thinking. We shall be as lenient as possible with our grading, and try to give as much part marks as possible in every subjective question.

Best wishes! Rock the exam.

Rules and Regulations

Submission of Answer sheets

1. The digital copy of the answer sheet has to be mailed to `stems.cssch@gmail.com`. The submissions should be sent strictly within the time limit.
2. You are allowed to scan/take pictures of your answer sheet for submission. You may use TeX if you wish to submit a soft copy of the solution as a pdf file.
3. Ensure that your submitted copy is clear and readable. Illegible submissions such as bad handwriting and unclear photographs will be disqualified.
4. Send the entire answer sheet in one single mail. Repeated mails of solutions will not be accepted.
5. Email back the answer sheet from the same email id you registered from (this is the same id that you received the question paper in).
6. Do NOT write your name or school/location in your answer sheet. Write your registration ID if you have it.

Marking Scheme

1. The question paper is divided in two parts. Objective and Subjective.
2. Each objective question has 4 options, and is worth 4 marks. Each option is either correct, or not. For each correct option that you choose, and for each incorrect option that you do not choose, you get 1 mark each. For example, say the correct answers to a problem are (a), (b), (c) and you choose (b), (c), (d). You get 1 point for not choosing (a), 2 points for choosing (b) and (c). So, you get 3 marks total.
3. You are not required to show your work for the objective part of the paper.
4. For Objective questions having choice, each problem has at least one correct answer: *however, there might be many.* Choose ALL that apply, unless indicated otherwise.
5. For the objective section, write your answers in the form of a tabular column where the rows denote the question number in order and the columns options *(a)* to *(d)* and tick the appropriate grids.For instance if you want to mark the answers (a) and (d) for question 1, and option $(b), (c)$ for question 12, then this is how the corresponding row would look like:

	(a)	(b)	(c)	(d)
(1)	✓			✓
		⋮		
(10)		✓	✓	

6. Each subjective problem is worth 12 marks each.
7. The subjective problems will be graded only if you score above a certain cut-off (which will be decided later) in the objective section of the paper.
8. For the subjective questions you need to formally show your reasoning. Marks will be awarded for partially correct solutions.
9. There is no negative marking.

Miscellaneous

1. Any form of plagiarism will lead to disqualification
2. For solving the problems, you are allowed to use the Internet and books as resources.
3. Write the answer clearly, in a legible way. Write formal proofs wherever necessary. Be clear with your reasoning.
4. You are not allowed to post/discuss the problems in any online forum within the exam time.
5. For any querries regarding the paper, please email `stemscsschool@gmail.com`. We shall respond to you as soon as possible. Note that the email mentioned above is only for clarifying doubts related to question paper. Technical queries must be emailed at `stems.tessellate@gmail.com` .

Objective Questions

Problem 1. *In the island of knaves and knights, knights always tell the truth and knaves always lie. Knaves and Knights are called types of the people in the island.*

An adventurer enters the island and meets two islanders, Bill and Joe. They have the following conversation.

Adventurer: What are your types?

Bill: We are of the same type.

Joe: We are of different types.

Can you infer the types of Bill and Joe?

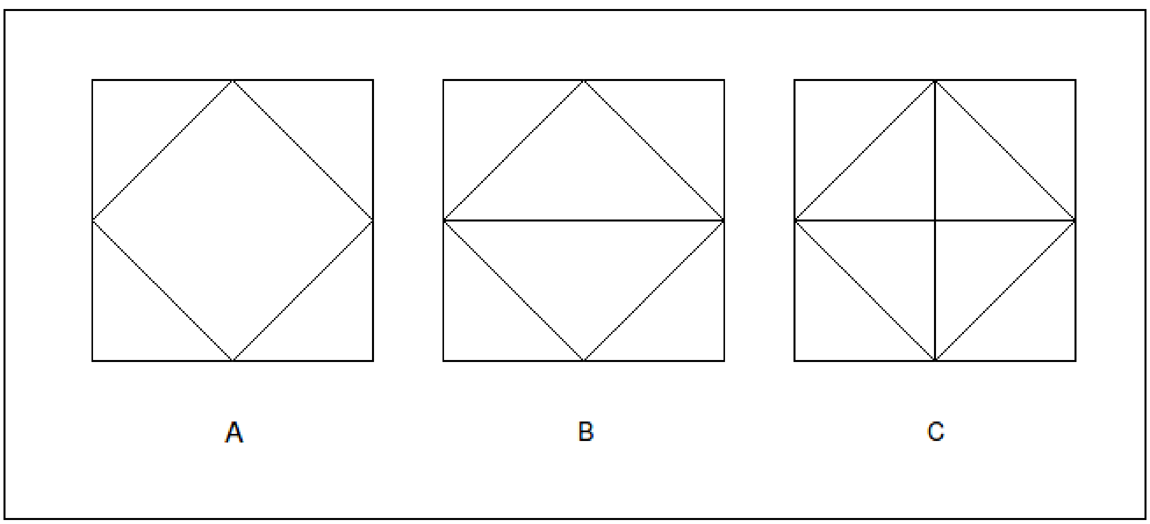
(a) *Bill is a knave, Joe is a knight*

(b) *Bill is a knight, Joe is a knave*

(c) *Both Bill and Joe are knights*

(d) *Both Bill and Joe are knaves*

Problem 2. *Consider the following figures:*



Consider the following rule

Rule: *You must draw a figure without lifting the pen and without retracing a line segment more than once.*

Which of these figures can be drawn following the rule?

(a) A

(b) B

(c) C

(d) None of them

Problem 3. *Suppose a and b are two bits. We define $a \oplus b$ as follows.*

$$0 \oplus 0 = 0$$
$$0 \oplus 1 = 1$$
$$1 \oplus 0 = 1$$
$$1 \oplus 1 = 0$$

Now, suppose there are three bits: a, b, c

How many (ordered) triplets (of bits) (a, b, c) satisfy

$$a \oplus b = b \oplus c = c \oplus a = 1$$

?

(a) 0

(b) 1

(c) 2

(d) 8

Problem 4. *A manager wants to hire an employee. There are 50 candidates, waiting outside in a queue.*

The manager employs the following algorithm to select a candidate.

```
Interview Candidate[1].
SET temporary_candidate TO Candidate[1]
FOR i = 2 TO 50:
  Interview Candidate[i]
  WITH probability 1/i:
    REPLACE temporary_candidate WITH Candidate[i]
HIRE temporary_candidate
```

Now consider the 1*st,* 25*th and the* 50*th candidate.*

Who is most likely to get hired?

(a) 1*st Candidate*

(b) 25*th Candidate*

(c) 50*th Candidate*

(d) *All of them are equally likely*

Problem 5. *Alice and Bob are playing a game.*

- *The game consists of moving a token from one node of the graph to another node to which there is a directed edge.*
- *Alice and Bob takes turns to move the token.*
- *Alice plays first.*
- *Initially, the token is placed on vertex 1, as indicated.*
- *When a person cannot play any more, he looses. And, the non-looser is said to have won.*

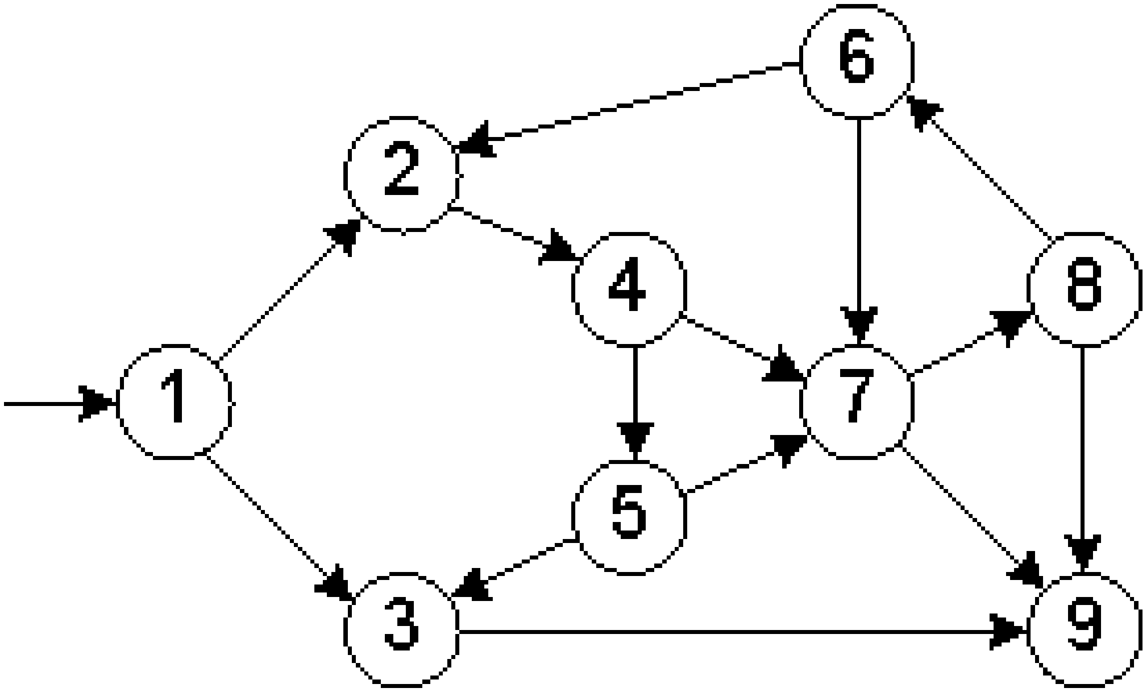
Assuming that both players play optimally, who has a winning strategy?

(a) *Alice*

(b) *Bob*

(c) *The game may continue indefinitely*

(d) *The game may end in a draw*



Here is an example play (not necessarily optimal):

- *Alice moves the token to* 3.
- *Bob moves the token to* 9.
- *Now, Alice cannot move any more. Hence, Bob wins.*

Problem 6. *You have n integers: $\{1, 2, \cdots n\}$. You divide this set in such a way that the absolute difference of the sum of the numbers in either set is minimized.*

For a given n, lets call this minimum possible value $f(n)$. Evaluate

$$\max_{n \leq 100} f(n)$$

(a) *1*

(b) *4*

(c) *50*

(d) *100*

Problem 7. *There are a total of 30 gold coins in three wooden boxes (but you do not know how many in each individual box). However, you know that one box has exactly 4 coins more than another box.*

For each box, you can ask for a number of coins from that box, of your choice. If there are at least that many coins in that box, then you get as many coins as you asked for. Otherwise, you get nothing from that box.

You must place all your demands simultaneuosly in the beginning.

What is the maximum number of coins that you can guarantee yourself to get?

(a) 10

(b) 11

(c) 12

(d) 13

Problem 8. 6 *people have* 6 *distinct pieces of a file, and they are linked over a network. Their goal is to make sure that every person gets all* 6 *pieces. To do this, each person can send a message to any other person communicating the pieces of the file he has collected so far.*

What is the minimum number of messages they could send in total so as to achieve this?

(a) 9

(b) 10

(c) 11

(d) 12

Problem 9.

$$k = \frac{512!}{256! \times 128! \times \cdots 2! \times 1!}$$

Statement I: 2^9 *divides* k.

Statement II: 2^{10} *divides* k.

(a) *Statement I is true, Statement II is true.*

(b) *Statement I is false, Statement II is true.*

(c) *Statement I is true, Statement II is false.*

(d) *Statement I is false, Statement II is false.*

Problem 10. *You are given two programs*

```
SET n := 1
SET ans  := 0

while (n <= m):
        SET w := n
        while (w <= m):
                SET ans := ans + w
                SET w := w + n
        SET n = n + 1

f(m) = ans
```

```
SET n := 1
SET ans := 0

while (n <= m):
        SET w := 1
        while (w <= n):
                if w divides n:
                        SET ans := ans + w
                SET w := w + 1
        SET n = n + 1

g(m) = ans
```

(a) $g(2k) = f(2k)$ *for all* k*,* k *being a natural number.*

(b) $g(2k) = f(2k+1)$ *for all* k*,* k *being a natural number.*

(c) $g(2k+1) = f(2k)$ *for all* k*,* k *being a natural number.*

(d) $g(2k-1) = f(2k-1)$ *for all* k*,* k *being a natural number.*

Subjective Questions

Problem 11. *A function $f(.,.)$ is defined as follows. If multiple rules apply, then the one listed first must be used to evaluate the function*

- *If* $i > j$*, then* $f(i,j) = f(i-1,j)$
- *If* $i \leq j$ *and* $i = 0$*,* $f(i,j) = 0$
- *Otherwise,* $f(i,j) = f(i-1,j) + i$

Given non-negative integers i and j, show that

$$f(i,j) = \frac{i(i+1)}{2}$$

Problem 12. *In a country, there are $2n$ cities, and they often have roads paved with yellow stones between them. However, if there are three cities a, b and c such that there is a road between a and b, and a road between b and c, there is no road between a and c, so as to reduce the country's expenditure on roads.*

Show that the country has at most n^2 roads.

Problem 13. *There is a 10,000 storied building and you have 9 identical Nokia phones. You believe that they are really strong and want to test them out. There is a floor such that if you drop a phone from that floor, it breaks (and cannot be used again), and for every floor below it, dropping them from them does not damage the phone at all. You need to figure out which floor this is.*

Show that you can figure out this floor by throwing phones a total of 27 times. If you cannot get this bound, show us the best bound you can get.

Problem 14. *A set of roads between m cities is called a* **cycle** *if there is a road from city 1 to 2, city 2 to 3, and so on, with a road from city m to 1.*

For some value of n, there is a country consisting of n cities and $n+3$ roads with a very nice property: If there are two cycles in this country, then they must have at least one city in common.

Construct this country, i.e, describe what its cities and roads must look like.

Problem 15. *You have been captured by the devil, but he proposes a game, in which if you win, you may go free. The game is as follows.*

- *The devil chooses a natural number k and gives you k sheets of paper.*
- *On every sheet, you are required to write the natural numbers from 1 to 2^k inclusive, and you may write on both sides of the sheet.*
- *The Devil now arranges the papers side by side, choosing a side for each of them, as they like.*
- *The Devil wins if he can arrange them in such a way that each of the numbers from 1 to 2^k is on the top side of at least one of the sheets, after he has laid them down.*

Who has a winning strategy?

For example, if k were just 1, then you could write 1 on one side of the sheet, and 2 on the other. That way, you'd win, since the Devil has no way to lay the sheets down in a way such that both 1 and 2 are visible.