TESSELLATE PRESENTS

STEMS

Scholastic Test of Excellence in Mathematical Sciences

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Physics Exam

Category C Sample Paper

October 14, 2018

Rules and Regulations

Marking Scheme

1. The question paper is divided in two parts: Objective and Subjective.
2. Each objective question is worth **2 point** and there is no negative marking.
3. You are not required to show your work for the objective part of the paper.
4. Each subjective problem is worth **10 points**.
5. For getting full credit in the subjective questions you need to give the detailed solutions. However, credit will also be awarded for partially correct solutions.
6. There is no negative marking in the subjective section as well.
7. **The subjective part will be graded only if you score above a certain cut-off (which will be decide later) in the objective section of the paper. However, for the final score, your total score (subjective + objective) will be taken into consideration.**

Miscellaneous

1. Any form of plagiarism will lead to disqualification
2. For solving the problems, you are allowed to use the Internet and books as resources.
3. Write the answer clearly, in a legible way. Write formal proofs wherever necessary. Be clear with your reasoning.
4. You are not allowed to post/discuss the problems in any online forum within the exam time.

Objective Questions

For **Problems 1-10**, each problem has **four** options, namely **(a), (b), (c), (d)**, of which ***only one*** is correct, **2 point** will be awarded for correctly answering a problem, **NO** negative marks shall be awarded for wrong answers/unattempted problems .

Problem 1. *Which of the following equation is the Euler-Lagrangian equation corresponding to the Lagrangian* $\mathcal{L} = \dot{x}\dot{y} - xy$*?*

(*a*) $\dot{y} = -x$, $\dot{x} = -y$

(*b*) $\ddot{x} = -x$, $\ddot{y} = -y$

(*c*) $\ddot{x} = -y$, $\ddot{y} = -x$

(*d*) $\dot{x} = -x$, $\dot{y} = -y$

Problem 2. *The root-mean square speed of molecules in an ideal gas of molar mass M at temperature T is(R denotes the ideal gas constant):*

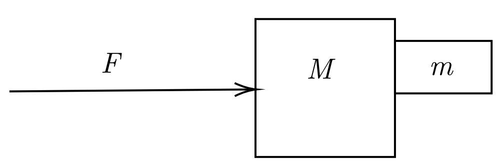
(*a*) *0*

(*b*) $\sqrt{\frac{RT}{M}}$

(*c*) $\sqrt{\frac{3RT}{M}}$

(*d*) $\frac{3RT}{M}$

Problem 3. *Two blocks of masses M and m are oriented as shown in the diagram. The block M moves on a frictionless surface, and the coefficient of static friction between the two blocks is $\frac{1}{2}$. What is the minimum amount of force F which must be applied on M so that the block m remains stationary with respect to the block M?*



(*a*) $2mg$

(*b*) $\frac{1}{2}(m + M)g$

(c) $\frac{mg}{2}$

(d) $2(m + M)g$

Problem 4. *Gauss' law would be invalid if*

(a) *there were magnetic monopoles*

(b) *the inverse-square law were not exactly true*

(c) *the velocity of light were not a universal constant*

(d) *None of the above*

Problem 5. *Let a and b denote position eigen states with eigenvalues a and b respectively. Let $a \neq b$. Compute the matrix element $< a \mid \hat{x} \mid b >$.*

(a) $\frac{a+b}{2}$

(b) a

(c) b

(d) 0

Problem 6. *Two ions 1 and 2 at fixed separation, with spin angular momentum operators S_1 and S_2 have the interaction Hamiltonian $H = -JS_1.S_2$, where $J > 0$. Values of S_1^2 and S_2^2 are fixed at $S_1(S_1+1)$ and $S_2(S_2+1)$ respectively. Which of the following is the energy of the ground state of system?*

(a) 0

(b) $-J[S_1(S_1 + 1) - S_2(S_2 + 1)]$

(c) $-(J/2)[(S_1 + S_2)(S_1 + S_2 + 1) - S_1(S_1 + 1) - S_2(S_2 + 1)]$

(d) $\frac{J}{2}\left[\frac{S_1(S_1+1)+S_2(S_2+1)}{S_1+S_2)(S_1+S_2+1)}\right]$

Problem 7. *Suppose there is a system with three energy levels $0, \epsilon$ and 8ϵ at temperature T. Calculate the average energy in the limit $T \longrightarrow 0$.*

(a) *0*

(b) 3ϵ

(c) 2.5ϵ

(d) 0.5ϵ

Problem 8. *A system consists of N weakly interacting systems with 2 internal quantum states with energies 0 and ϵ. Internal energy for this system at the absolute temperature T is equal to:*

(a) $N\epsilon$

(b) $\frac{3}{2}NKT$

(c) $N\epsilon e^{-\epsilon/KT}$

(d) $\frac{N\epsilon}{e^{\epsilon/KT}+1}$

Problem 9. *Wave function for identical fermions is anti-symmetric under particle interchange. Which of the following is a consequence of this property?*

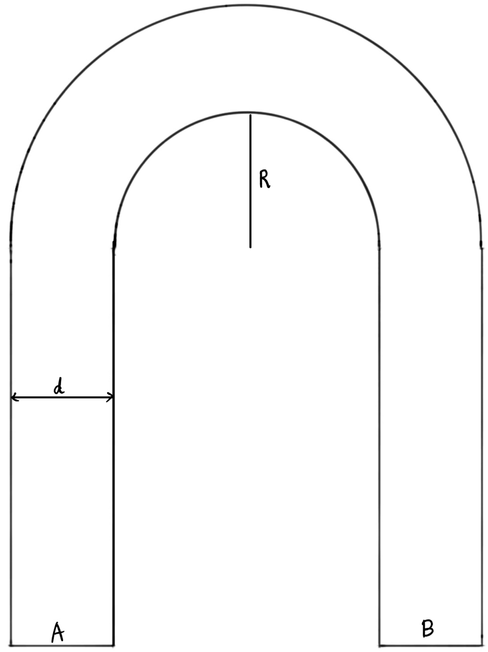
(a) *Pauli Exclusion Principle*

(b) *Bohr Correspondence Principle*

(c) *Heisenberg Uncertainty Principle*

(d) *Bose-Einstein Condensation*

Problem 10. *A glass rod of rectangular cross-section is bent into the shape of a horse-shoe as shown in the figure below. Suppose a beam of light is falling perpendicular on the surface A. Then what is the minimum value of $\frac{R}{d}$ for which the whole beam of light emerges through the surface B, given the refractive index of glass to be 1.5.*



(a) *1.5*

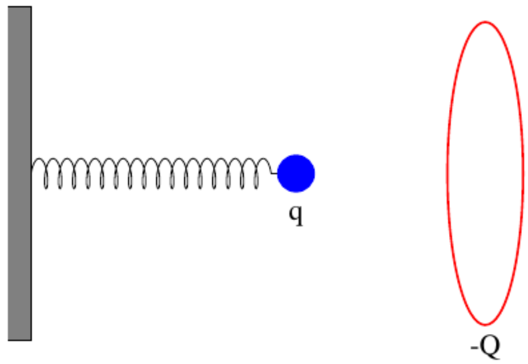
(b) *2*

(c) *2.5*

(d) *3*

Subjective Problems

Problem 1. *Consider a particle of mass m and charge q attached to a spring with spring constant k and constrained to move along x-axis. Suppose at equilibrium the particle is at origin. Now a uniformly charged ring of radius R with charge $-Q$ and x-axis as its axis is placed such that the center of the ring is located at $(L, 0, 0)$. (Assume $q, Q > 0$) Find the new equilibrium point of the particle.*



Problem 2. *Obtain the energy levels of the two particle system, each of mass m given by*

$$H = \frac{p_1^2 + p_2^2}{2m} + \frac{mw^2(x_1^2 + x_1^2)}{2} + \lambda x_1 x_2$$

Find the energy levels and write down the wave function of the first excited state in terms of creation and annihilation operators and the vacuum state.

Problem 3. *Given Hamiltonian $H(q, p)$, write down the Taylor series for $q(t)$ in terms of Poisson brackets, given the initial condition $q(0) = q_0$ and $p(0) = p_0$.*